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Massive-Evolution Effects on Charmonium Hadroproduction

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Abstract

The fragmentation functions $D_{a\to H}(x,\mu^2)$ of a heavy hadron H, with mass m_H, satisfy the phase-space constraint $D_{a\to H}(x,\mu^2)=0$ for $x<m_H^2/\mu^2$, which is violated by the naive μ^2 evolution equations. Using appropriately generalized μ^2 evolution equations, we reconsider the inclusive hadroproduction of prompt J/ψ mesons with high transverse momenta in the framework of the factorization formalism of nonrelativistic quantum chromodynamics, and determine the resulting shifts in the values of the leading colour-octet matrix elements, which are fitted to data from the Fermilab Tevatron.

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In the framework of the QCD-improved parton model, the inclusive production of single hadrons is described by using fragmentation functions, $D_{a\to h}(x,\mu^2)$. The value of $D_{a\to h}(x,\mu^2)$ corresponds to the probability for a parton a which comes out of the hard-scattering process to form a jet which contains a hadron h carrying the longitudinal-momentum fraction $x=(p_h^0+p_h^3)/(p_a^0+p_a^3)$, where p_a and p_h are the four-momenta of a and h in the infinite-momentum frame. Here, μ is the fragmentation scale, which is typically chosen to be of the order of the centre-of-mass energy $\sqrt{s}$ (transverse momentum p_T of h) in lepton-lepton (lepton-hadron and hadron-hadron) collisions. At next-to-leading order (NLO), μ is identified with the mass scale at which the collinear singularities associated with the outgoing parton a are factorized. In the case of a light hadron, with mass $m_h \ll \mu$, the μ^2 dependence of $D_{a\to h}(x,\mu^2)$ is determined by the timelike Altarelli-Parisi evolution equations,

$$\frac{\mu^2\partial}{\partial\mu^2}D_{a\to h}(x,\mu^2)=\sum_b\int_x^1\frac{dy}{y}P_{a\to b}^{(T)}(y,\mu^2)D_{b\to h}\left(\frac{x}{y},\mu^2\right), \tag{1}$$

where $P_{a\to b}^{(T)}(x,\mu^2)$ are the timelike $a\to b$ splitting functions. Ready-to-use expressions for $P_{a\to b}^{(T)}(x,\mu^2)$ through NLO are collected in the Appendix of Ref. [1].[1]

In Eq. (1), x may, in principle, be arbitrarily low for any value of μ. In the case of a heavy hadron H, with mass $m_H \lesssim \mu$, this conflicts with the fact that the fragmentation process is only allowed by kinematics if the virtuality p_a^2 of the fragmenting parton a satisfies the condition $p_a^2 > m_H^2/x$. This may be understood as follows. Consider the general situation where a fragments into n particles, with masses m_i and four-momenta p_i, and choose the coordinate system so that $p_a^\mu=(p_a^0,0,0,p_a^3)$. Defining the longitudinal-momentum fractions as $x_i=(p_i^0+p_i^3)/(p_a^0+p_a^3)$, we then have

$$p_a^2=\left(p_a^0+p_a^3\right)\sum_{i=1}^n\left(p_i^0-p_i^3\right)=\sum_{i=1}^n\frac{m_i^2+p_{T,i}^2}{x_i}, \tag{2}$$

where $p_{T,i}=\sqrt{(p_i^1)^2+(p_i^2)^2}$ are the intrinsic transverse momenta. If all final-state particles, except for H, are light and the intrinsic transverse momenta are neglected, as is usually done in the parton model, then it follows that $p_a^2\ge m_H^2/x$. If we identify $\mu^2=p_a^2$, then this phase-space constraint leads to the condition $D_{a\to H}(x,\mu^2)=0$ for $x<m_H^2/\mu^2$.

In order to properly implement this condition, Eq. (1) must be generalized. Such a generalization was proposed in Refs. [2, 3] for the μ^2 evolution of $D_{g\to H}(x,\mu^2)$ generated by the $g\to g$ splitting. It is straightforward to extend this formalism to also include the fragmentation of the other partons and the nondiagonal evolution effects. This leads us to the ansatz

$$D_{a\to H}(x,\mu^2)=\sum_b\int_{m_H^2}^{\mu^2}\frac{dq^2}{q^2}\int_x^1\frac{dy}{y}G_{a\to b}(y,q^2;\mu^2)d_{b\to H}\left(\frac{x}{y},q^2\right), \tag{3}$$

[1] There is an obvious typographical error in the published version of Ref. [1], which was absent in the preprint version thereof. In the line before the last of Eq. (17), $\ln\ln(1-x)$ should be replaced by $\ln(1-x)$.

where $G_{a\to b}(y,q^2;\mu^2,)$ corresponds to the probability for the parton a, with virtuality μ^2, to emit a parton b, with longitudinal-momentum fraction y and virtuality q^2, and $d_{b\to H}(z,q^2)$ to the one for parton b to subsequently decay to a hadron H, with longitudinal-momentum fraction z relative to b. The μ^2 dependence of $G_{a\to b}(y,q^2;\mu^2)$ is determined by the evolution equation

$$\frac{\mu^2\partial}{\partial\mu^2}G_{a\to b}(y,q^2;\mu^2)=\sum_c\int_y^1\frac{dz}{z}P^{(T)}_{a\to c}(z,\mu^2)G_{c\to b}\left(\frac{y}{z},q^2;z\mu^2\right), \tag{4}$$

which is similar to Eq. (1), except that the virtuality of the intermediate parton c is taken to be $z\mu^2$ instead of μ^2.[2] Furthermore, $G_{a\to b}(y,q^2;\mu^2)$ satisfies the boundary condition

$$G_{a\to b}(y,\mu^2;\mu^2)=\delta_{ab}\delta(1-y). \tag{5}$$

According to the above argument, $G_{a\to b}(y,q^2;\mu^2)$ is subject to the phase-space constraint $G_{a\to b}(y,q^2;\mu^2)=0$ for $y<q^2/\mu^2$. For the same reason, we have $d_{b\to H}(z,q^2)=0$ for $z<m_H^2/q^2$. From Eq. (3) it hence follows that $D_{a\to H}(x,\mu^2)=0$ for $x=yz<m_H^2/\mu^2$ as desired.

In the perturbative calculation of the fragmentation function $D_{a\to H}\left(x,\mu_0^2\right)$ at the initial scale μ_0, with $\mu_0\gtrsim m_H$, $d_{a\to H}(x,q^2)$ acts as a source density, in the sense that

$$D_{a\to H}\left(x,\mu_0^2\right)=\int_{m_H^2}^{\mu_0^2}\frac{dq^2}{q^2}d_{a\to H}(x,q^2), \tag{6}$$

which follows from Eq. (3) by approximating $G_{a\to b}(y,q^2;\mu^2)$ with the aid of Eq. (5). In practice, the upper bound of integration in Eq. (6) is taken to be infinity because the integrand rapidly falls off with increasing q^2 [4, 5].

Differentiating Eq. (3) with respect to $\ln\mu^2$ and substituting Eqs. (4) and (5) on the right-hand side, we may eliminate $G_{a\to b}(y,q^2;\mu^2)$ and thus obtain a single set of inhomogenious integro-differential evolution equations for $D_{a\to H}(x,\mu^2)$, namely,

$$\frac{\mu^2\partial}{\partial\mu^2}D_{a\to H}(x,\mu^2)=d_{a\to H}(x,\mu^2)+\sum_b\int_x^1\frac{dy}{y}P^{(T)}_{a\to b}(y,\mu^2)D_{b\to H}\left(\frac{x}{y},y\mu^2\right), \tag{7}$$

which is to be solved imposing the boundary condition $D_{a\to H}\left(x,m_H^2\right)=0$. Evidently, the solutions of Eq. (7) satisfy the condition $D_{a\to H}(x,\mu^2)=0$ for $x<m_H^2/\mu^2$ as they should. Notice that Eq. (7) carries over to NLO as it stands. One just needs to include the NLO corrections to $d_{a\to H}(x,\mu^2)$ and $P^{(T)}_{a\to b}(y,\mu^2)$ and to employ the two-loop formula for the strong coupling constant $\alpha_s(\mu^2)$.

We now explore the phenomenological implications of the modified μ^2 evolution for the case of prompt J/ψ-meson production via fragmentation, $H=J/\psi$, in the framework of the factorization formalism of nonrelativistic QCD (NRQCD) [6]. At the starting scale μ_0, the leading contributions arise from the fragmentation processes $a\to c\bar{c}[n]$

[2] Here, we deviate from Eq. (4) of Ref. [2], which refers to the evolution in q^2.

Table 1: Values of the leading J/ψ colour-octet matrix elements (in units of 10^{-3} GeV^3) and of r resulting from the LO and HO-improved fits to the CDF data [13] with naive and modified μ^2 evolution.

	naive		modified	
	LO	HO	LO	HO
$\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$	4.58 ± 0.73	2.58 ± 0.42	5.01 ± 0.79	2.86 ± 0.46
$M_r^{J/\psi}$	104 ± 10	19.1 ± 2.8	99.9 ± 9.8	17.0 ± 2.7
r	3.44	3.53	3.43	3.52
χ^2_{DF}	0.52	0.52	0.52	0.41

with $a = g, c, \bar{c}$ and $[n] = [\underline{1}, {}^3S_1], [\underline{8}, {}^3S_1]$, where $\underline{1}$ and $\underline{8}$ label colour-singlet and colour-octet states, respectively, and the spectroscopic notation ${}^{2S+1}L_J$ indicates the spin S, the orbital angular momentum L, and the total angular momentum J. The corresponding fragmentation functions $D_{a\to J/\psi}(x, \mu_0^2)$ may be found in Refs. [4, 7] and their source densities $d_{a\to J/\psi}(x, q^2)$ may be gleaned from Ref. [4]. The fragmentation functions for $a = u, \bar{u}, d, \bar{d}, s, \bar{s}$ are generated via the μ^2 evolution and coincide. Furthermore, we have $D_{c\to J/\psi}(x, \mu^2) = D_{\bar{c}\to J/\psi}(x, \mu^2)$, so that we only need to distinguish the three cases $a = g, c, u$. In Figs. 1a-c, we study the x dependences of $D_{a\to J/\psi}(x, \mu^2)$ at $\mu^2 = 300$ GeV^2 for $a = g, c, u$, respectively, comparing the the naive (dashed lines) and modified (solid lines) μ^2 evolutions to leading order (LO). We adopt the LO color-singlet matrix element from Ref. [8] and the LO colour-octet ones from Table 1, selecting those which refer to the naive μ^2 evolution. The μ^2 evolutions are performed iteratively in x space. The technicalities are explained for the naive and modified μ^2 evolutions in Refs. [7, 9], respectively. As is well known [10], the naive μ^2 evolution equations (1) break down in the low-x region due to the presence of large logarithms of $1/x$ on their right-hand sides. The most dramatic consequence of this deficiency is an unphysical divergence in the gluon multiplicity at low x. This directly triggers the singular low-x behaviour of $D_{g\to J/\psi}(x, \mu^2)$, which is exhibited by the dashed line in Fig. 1a. Via nondiagonal evolution effects, this feature also feeds into the quark fragmentation functions, as may be seen from Figs. 1b and c. By contrast, the results based on the modified μ^2 evolution are devoid of such low-x divergences. As per construction, they vanish for $x < m_{J/\psi}^2/\mu^2 = 0.032$. On the other hand, they merge with the naive evaluation as x approaches unity because the impact of the phase-space constraint then fades out. Both features serve as a welcome check for our numerical analysis. Further evidence for the physical soundness of the modified μ^2 evolution comes from a recent phenomenological analysis of Z-boson decay into prompt J/ψ mesons via fragmentation [11], where the large logarithms of the fixed-order calculation were resummed by using a Monte Carlo cascade model. Except very close to the threshold region, the results thus generated were found to nicely agree with those obtained from the modified μ^2 evolution.

At present, one of the most important applications of the J/ψ-meson fragmentation functions is to describe the inclusive hadroproduction of prompt J/ψ mesons with high transverse momenta in $p\bar{p}$ collisions at the Fermilab Tevatron, with $\sqrt{s} = 1.8$ TeV. In fact, fits [8, 12] to the latest data on $p\bar{p} \to J/\psi + X$ taken by the CDF Collaboration [13] allow one to place stringent constraints on the leading colour-octet matrix elements, $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$, $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^1S_0] \right\rangle$, and $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3P_J] \right\rangle$, with $J = 0, 1, 2$, in the framework of the NRQCD factorization formalism [6]. Specifically, the data in the upper p_T range are especially sensitive to $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$, while the low-$p_T$ data essentially fix the linear combination

$$M_r^{J/\psi} = \left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^1S_0] \right\rangle + \frac{r}{m_c^2} \left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3P_0] \right\rangle, \tag{8}$$

where r is to be chosen in such a way that the superposition of these two channels is insensitive to precisely how they are weighted relative to each other. Due to heavy-quark spin symmetry, the multiplicity relation $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3P_J] \right\rangle = (2J+1) \left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3P_0] \right\rangle$ is approximately satisfied. Adhering to the analysis of Ref. [8], we now investigate by how much the fit values for $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$ and M_r are shifted if we pass from the naive to the modified μ^2 evolution The philosophy advocated in Ref. [8] is adopt the fusion picture, where the $c\bar{c}$ bound state is formed within the primary hard-scattering process, in the low-p_T regime and the fragmentation picture, where the $c\bar{c}$ bound state is created from a single parton which is close to its mass shell, in the high-p_T regime. We first redo the LO and higher-order-improved (HO) analyses of Ref. [8], which are based on slightly obsolete parton density functions, using the latest LO and NLO MRST sets [14]. We then repeat these calculations with the modified μ^2 evolution. The resulting LO and HO values for $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$ and $M_r^{J/\psi}$ are summarized in Table 1 together with the corresponding values of χ^2 per degree of freedom, χ^2_{DF}. The LO and HO values for $\left\langle \mathcal{O}^{J/\psi}[\underline{1}, {}^3S_1] \right\rangle$ remain the same as in Ref. [8]. We observe that the effect of the modified μ^2 evolution is to increase $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$ and to decrease $M_r^{J/\psi}$. This may be understood from Figs. 1a–c by observing that the use of the modified μ^2 evolution leads to a reduction of the fragmentation functions at low and intermediate values of x. This must be compensated by an increase of $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$ in order to match the high-p_T data, which are described in the fragmentation picture. On the other hand, the use of this increased value of $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$ in the fusion picture must in turn be compensated by a reduction of $M_r^{J/\psi}$ in order to fit the low-p_T data. Within each order of perturbation theory, the shifts in $\left\langle \mathcal{O}^{J/\psi}[\underline{8}, {}^3S_1] \right\rangle$ and $M_r^{J/\psi}$ due to the change of the evolution mode are relatively modest, comparable to the errors on these quantities. This may be understood by considering the average x values which are probed by $p\bar{p} \to J/\psi + X$ via fragmentation. At the data point of highest p_T, with $p_T \approx 18$ GeV, we have $x = 0.73 \pm 0.14$ in the colour-singlet channel, $x = 0.90 \pm 0.12$ in the colour-octet channel, and $x = 0.90 \pm 0.12$ for their combination. As is evident from Figs. 1a–c, the massive-evolution effect is not yet pronounced at such high x values.

In conclusion, the implementation of the phase-space constraint for heavy hadrons in the μ^2 evolution leads to a significant reduction of their fragmentation functions at low

and intermediate values of the longitudinal-momentum fraction x. However, current determinations of the leading colour-octet matrix elements for the J/ψ meson from Tevatron data of $p\bar{p} \to J/\psi + X$ are only modestly affected by this because the x values which are typically probed in this reaction are rather high.

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References

[1] J. Binnewies, B.A. Kniehl and G. Kramer, Z. Phys. C 76 (1997) 677.

[2] E. Braaten, M.A. Doncheski, S. Fleming and M.L. Mangano, Phys. Lett. B 333 (1994) 548.

[3] M.L. Mangano and A. Petrelli, Int. J. Mod. Phys. A 12 (1997) 3887.

[4] E. Braaten and T.C. Yuan, Phys. Rev. Lett. 71 (1993) 1673; Phys. Rev. D 52 (1995) 6627; E. Braaten, K. Cheung and T.C. Yuan, Phys. Rev. D 48 (1993) 4230.

[5] E. Braaten and T.C. Yuan, Phys. Rev. D 50 (1994) 3176; T.C. Yuan, Phys. Rev. D 50 (1994) 5664; E. Braaten and Y.-Q. Chen, Phys. Rev. D 55 (1997) 2693.

[6] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D 51 (1995) 1125; 55 (1997) 5855 (E).

[7] B.A. Kniehl and G. Kramer, Phys. Rev. D 56 (1997) 5820.

[8] B.A. Kniehl and G. Kramer, Eur. Phys. J. C 6 (1999) 493.

[9] L. Zwirner, Diploma thesis, University of Munich (October 1998).

[10] A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rep. 100 (1983) 201.

[11] P. Ernström, L. Lönnblad and M. Vänttinen, Z. Phys. C 76 (1997) 515.

[12] P. Cho and A.K. Leibovich, Phys. Rev. D 53 (1996) 150; 53 (1996) 6203; M. Beneke and M. Krämer, Phys. Rev. D 55 (1997) 5269.

[13] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 79 (1997) 578.

[14] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Phys. Lett. B 443 (1998) 301; Report No. DTP/99/64, OUTP/9931P, RAL-TR-1999-047 and hep-ph/9907231 (July 1999).

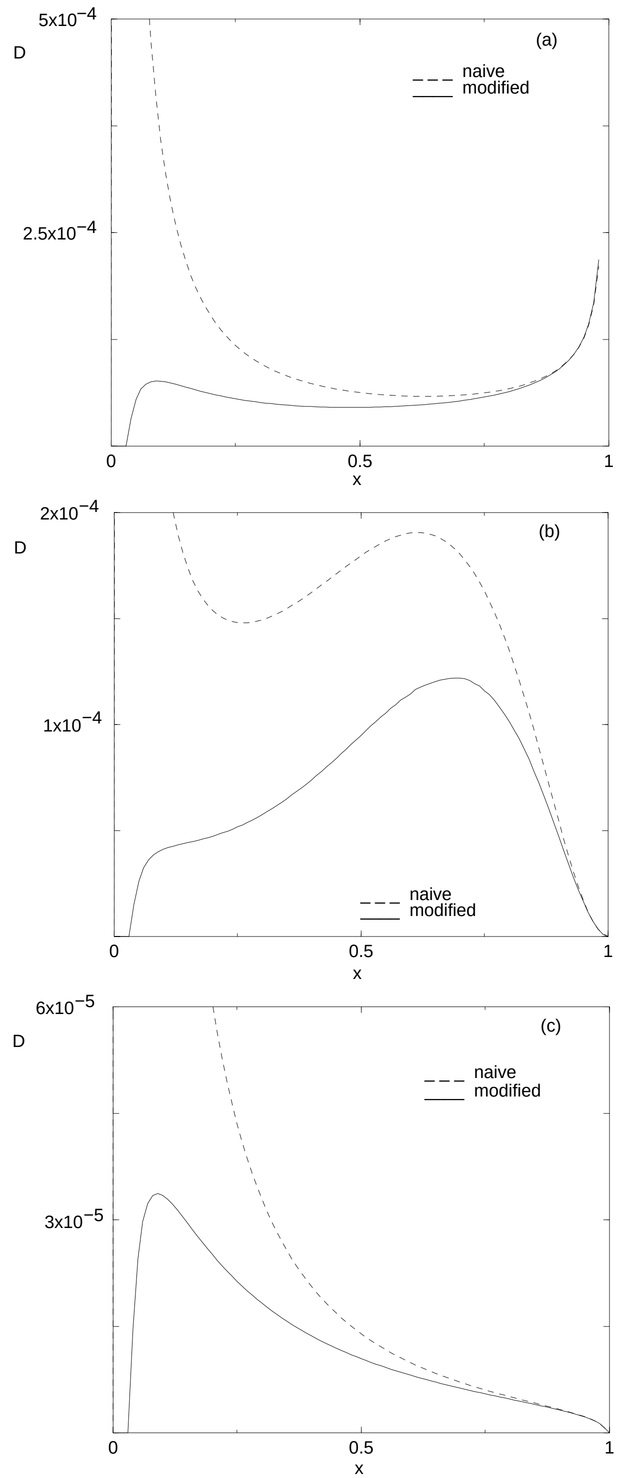

Figure 1: $D_{a\to J/\psi}(x,\mu^2)$ to LO at $\mu^2 = 300$ GeV^2 as a function of x for (a) $a = g$, (b) $a = c$, and (c) $a = u$. The results obtained with the modified μ^2 evolution (solid lines) are compared with those obtained with the naive μ^2 evolution (dashed lines).